CBSE Board
Class X Mathematics
Sample Paper 3 (Standard)

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. This question paper contains **two parts** A and B.
2. Both **Part A** and **Part B** have internal choices.

Part – A:

1. It consists **two sections** - I and II.
2. **Section I** has **16 questions** of **1 mark** each. Internal choice is provided in **5 questions**.
3. **Section II** has **4 questions** on **case study**. Each case study has **5 case-based sub-parts**. An examinee is to attempt any **4 out of 5 sub-parts**. Each subpart carries **1 mark.**

Part – B:

1. It consists **three sections** – III, IV and V
2. **Section III: Question No 21 to 26** are **Very short answer** Type questions of **2 marks** each.
3. **Section IV: Question No 27 to 33** are **Short Answer Type** questions of **3 marks** each.
4. **Section V: Question No 34 to 36** are **Long Answer Type** questions of **5 marks** each.
5. Internal choice is provided in **2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks**

Part A

Section I

Section I has 16 questions of 1 mark each.

(Internal choice is provided in 5 questions)

1. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, what is the other number?

OR

Express $0.\overline{8}$ as a fraction in simplest form.

2. If the mean of a data is 27 and its median is 33 then, find the value of mode.

3. Find $\sqrt{\frac{1+\sin A}{1-\sin A}}$.

4. If $\tan\theta = \frac{a}{b}$ then what is the value of $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$.

5. What is the value(s) of p, if the distance between the points A(4, p) and B(1, 0) is 5?

OR

A is a point on y-axis at a distance of 4 units from x-axis, lying below x-axis, then what are the coordinates of point A?

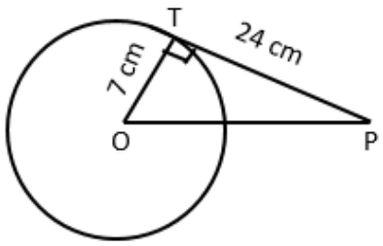
6. In ΔABC and ΔDEF, we have $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$ then find the value of $\frac{ar(\Delta ABC)}{ar(\Delta DEF)}$.

OR

If $\frac{AD}{DB} = \frac{4}{7}$ and AE = 6 cm where D and E are points on the sides AB and AC respectively of triangle ABC such that DE || BC. Find EC.

7. For an event E, what is the value of P(E) + P(not E)?

8. In a circle of radius 7 cm, tangent PT is drawn from a point P such that PT = 24 cm. If O is the centre of the circle, then find the length of OP.



9. Find the sum of first n even natural numbers.

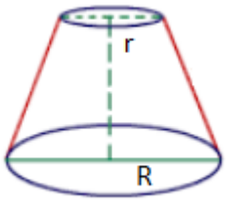
OR

Write the next term of the AP $\sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

10. Find the ratio of the sum and product of the roots of the equation $7x^2 - 12x + 18 = 0$.

11. Find the value of k for which the system of equations 3x + 5y = 0 and kx + 10y = 0 has a non-zero solution.

12. Write the formula for volume of the given figure.



13. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then find the value of k.

14. If the product of two zeros of the polynomial $f(x) = 2x^3 + 6x^2 - 4x + 9$ is 3, then find its third zero.

15. From the letters of the word "MOBILE", a letter is selected. What is the probability that the letter is a vowel?

16. If PT is a tangent at T to a circle whose centre is O and OP = 17 cm, OT = 8 cm, find the length of the tangent segment PT.

OR

Two concentric circles are of radius 30 cm and 18 cm. Find the length of the chord of the larger circle which touches the smaller circle.

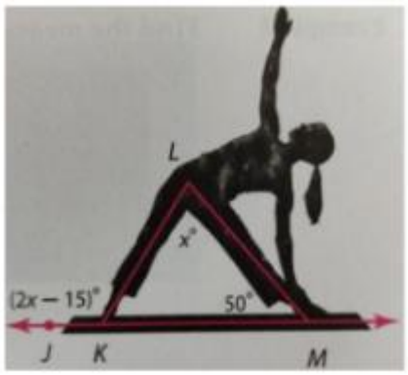
Section II

(Q 17 to Q 20 carry 4 marks each)

Case study based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. Case Study based-1
Types of angles and angle sum property of a triangle

It is 7:00 am!
Shikha rolls out her yoga mat and starts her warm up session with stretching and bending. Anaya her daughter is sitting nearby, observing her mother's daily ritual. Anaya takes a picture of her mother while she was in a yoga posture and label it as shown.



(a) Angles ∠LKM and ∠JKL are called as?
(i) Linear Pair of angles
(ii) Vertically opposite angles
(iii) Complementary angles
(iv) Corresponding angles

(b) Find m∠LKM.
(i) 195° – x
(ii) 185° – 2x
(iii) 195° – 2x
(iv) 185° – x

(c) Find m∠KLM.
(i) 115°
(ii) 65°
(iii) 50°
(iv) 180°

(d) Which of the following is true for △LKM?
(i) △LKM is an equilateral triangle.
(ii) △LKM is an isosceles triangle.
(iii) △LKM is a right angle triangle.
(iv) All of the above

(e) What is the measurement of the ∠LKJ?
(i) 115°
(ii) 65°
(iii) 50°
(iv) 180°

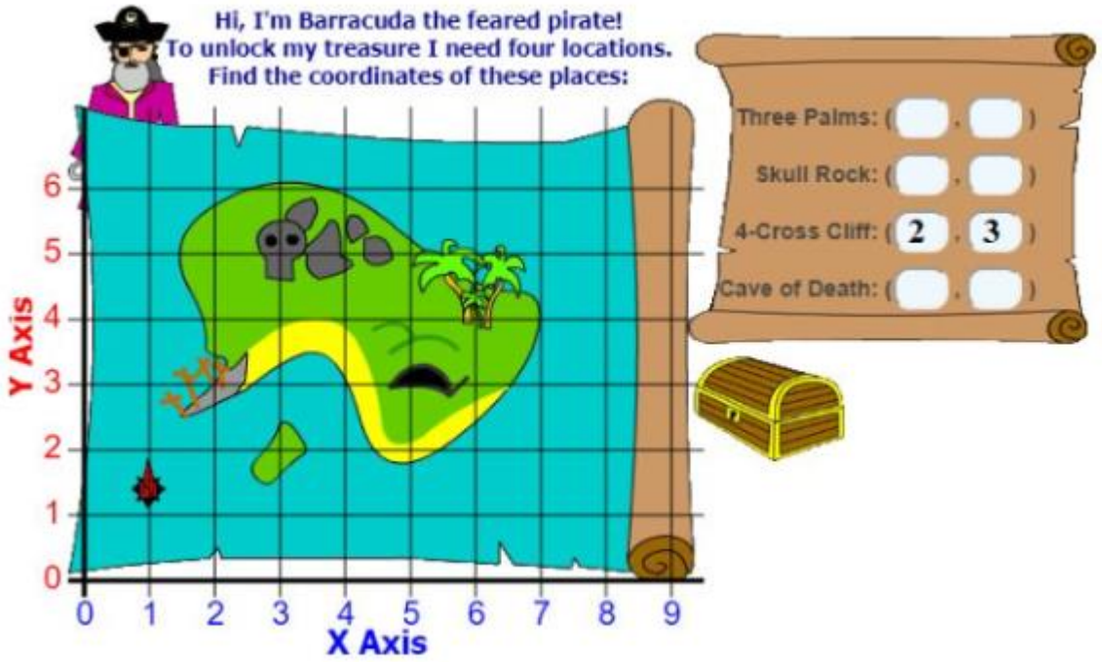
18. Case Study Based- 2
THE TREASURE ISLAND

Understanding Graphs:

On the graph sheet, a point is located using a pair of numbers such as (x, y)

- The first number 'x' shows the horizontal distance of the point (i. e left or right) on the horizontal line.
- The second number 'y' shows the vertical distance of the point (i. e up of down) right) on the vertical line.
- The point where X – axis and Y – axis cross each other at 90° called the Origin denoted by (0, 0).
- Clearly the X – axis and Y – axis divide the plane is known as Cartesian plane.
- We measure everything on the Cartesian plane with respect to Origin.

Rita and Renu are playing a board game of Treasure Island.



(a) The coordinate of CAVE of DEATH
- (i) (3, 5)
- (ii) (3, 3)
- (iii) (5, 5)
- (iv) (5, 3)

(b) The coordinate of THREE PALMS
- (i) (6, 3)
- (ii) (3, 6)
- (iii) (5, 2)
- (iv) (9, 5)

(c) The distance between FOUR CROSS CLIFF and the CAVE of DEATH is
- (i) 3 units
- (ii) 5 units
- (iii) 2 units
- (iv) None of these

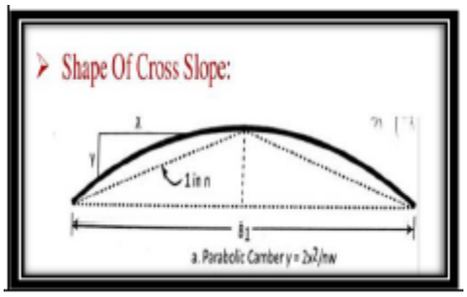
(d) What is the distance of SKULL ROCK from x – axis?
- (i) 3 units
- (ii) 5 units
- (iii) 2 units
- (iv) None of the

(e) The mid – point of CAVE of DEATH and THREE PALMS is
- (i) (5.5, 3.5)
- (ii) (5, 3)
- (iii) (3.5, 5.5)
- (iv) (3, 5)

19. Case Study Based- 3

Applications of Parabolas-Highway Overpasses/Underpasses

A highway underpass is parabolic in shape.

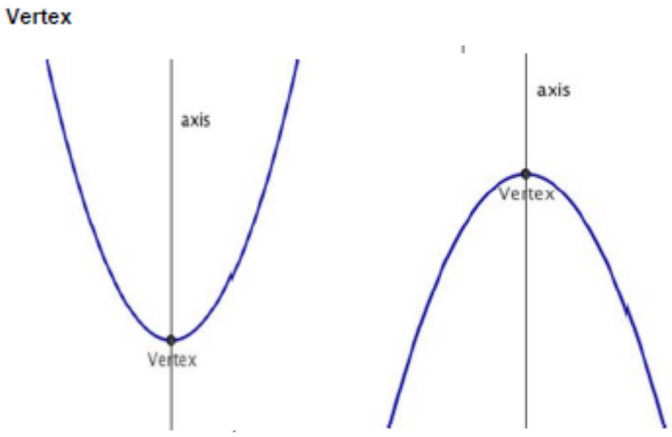


Parabola

A parabola is the graph that results from $p(x)=ax^2+bx+c$

Parabolas are symmetric about a vertical line known as the Axis of Symmetry.

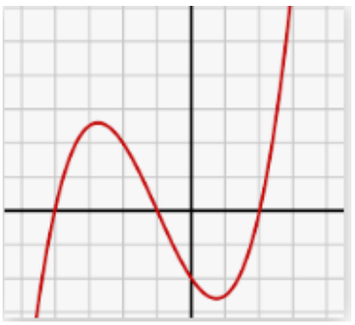
The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the



(a) If the highway overpass is represented by $x^2 - 18x + 81$. Then its zeroes are

(i) (9, –9) (ii) (9, 9) (iii) (–9, –9) (iv) (–9, 9)

(b)



Zeroes of a polynomial can be expressed graphically.

Number of zeroes of the given polynomial(refer image) is equal to

(i) one

(ii) two

(iii) three

(v) four

(c) The degree of a quadratic polynomial

(i) 0

(ii) 1

(iii) 2

(vi) 3

(d) The representation of Highway Underpass whose zeroes are 6 and 3 is

(i) $x^2 - 9x + 18$

(ii) $x^2 - 12x + 36$

(iii) $x^2 + 18x + 9$

(vii) $x^2 - 3x + 6$

(e) The number of zeroes that polynomial $f(m) = m^2$ can have is:

(i) 1

(ii) 2

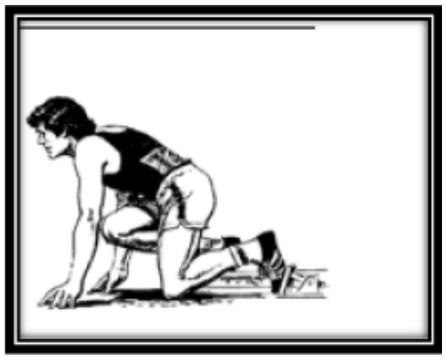
(iii) 0

(iv) 3

20. Case Study Based- 4

110m RACE

A stopwatch was used to find the time that it took a group of students to run 110m.



Time(in sec)	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of students	7	10	15	5	3

(a) Estimate the mean time taken by a student to finish the race.
  (i) 54.6
  (ii) 63.5
  (iii) 43.5
  (iv) 50.5

(b) What will be the lower limit of the modal class?
  (i) 20
  (ii) 40
  (iii) 60
  (iv) 80

(c) Which of the following are measures of Central Tendency?
  (i) Mean
  (ii) Median
  (iii) Mode
  (iv) All of the above

(d) The sum of upper limits of median class and modal class is
  (i) 60
  (ii) 120
  (iii) 80
  (iv) 160

(e) How many students finished the race within 1 min?
  (i) 18
  (ii) 37
  (iii) 17
  (iv) 8

Part B

All questions are compulsory. In case of internal choices, attempt any one.

Section III

(Q 21 to Q 26 carry 2 marks each)

21. In two concentric circles, the radius of the inner circle is 5 cm. A chord of length 24 m of the outer circle becomes a tangent to the inner circle. Find the radius of the larger circle.

22. How many solid spheres of diameter 6 cm are required to be melted to form a solid metal cylinder of height 45 cm and diameter 4 cm?

OR

Three cubes whose edge measures 3 cm, 4 cm and 5 cm respectively to form a single cube. Find its edge.

23. The sides of a certain triangle are 9 cm, 18 cm, and 16 cm. Determine whether these sides will form a right triangle or not?

OR

Corresponding sides of two triangles are in the ratio 2:3. If the area of the smaller triangle is 48 cm^2, determine the area of the larger triangle.

24. An umbrella has 10 ribs which are equally spaced. Assuming the umbrella to be a flat circle of radius 40 cm, find the area between two consecutive ribs of the umbrella.

25. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. Find the distance of the car from the tower (in metres).

26. A box contains 20 cards numbered from 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is

1. Divisible by 2 and 3

2. A prime number

Section IV

(Q 27 to Q 33 carry 3 marks each)

27. If the roots of the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal then prove that $2a = b + c$.

28. Prove that $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 89^\circ = 1$

29. Show that $6 + \sqrt{3}$ is irrational.

OR

Prove that $\sqrt{5}$ is an irrational number.

30. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield	50 – 55	55 – 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a 'more than' type distribution and draw ogive.

31. Prove that: $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\,\mathrm{cosec}\,\theta$

OR

Without using trigonometric table, find the value of $\frac{\cos 70°}{\sin 20°} + \frac{\cos 59°}{\sin 31°} - 8\sin^2 30°$

32. Find three terms of an A.P. whose sum is 3 and product is −8.

33. If mean of the following data is 86, then what is the value of p?

Wages (in Rs.)	50-60	60-70	70-80	80-90	90-100	100-110
No. of workers	5	3	4	p	2	13

Section V

(Q 34 to Q 36 carry 5 marks each)

34. Construct a triangle, the lengths of whose sides are 5 cm, 6 cm and 7 cm. Now construct another triangle whose sides are $\frac{5}{7}$ times the corresponding sides of the first triangle.

35. The angle of elevation of a cloud from a point 60 metres above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud.

OR

The angle of elevation of a cloud from a point 'h' m above a lake is α and the angle of depression of its reflection in the lake is β. Prove that height of the cloud is $\frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$.

36. A lead pencil consists of a wood cylinder with a solid cylinder of graphite fitted into it. The diameter of the pencil is 7 mm. The diameter of the graphite is 1 mm and length of the pencil is 10 cm. Calculate the weight of whole pencil in grams if the density of the wood is 0.6 gm/ cm^3 and of graphite 2.3 gm/ cm^3.

CBSE Board
Class X Mathematics
Sample Paper 3 (Standard) – Solution

Time: 3 hrs **Total Marks: 80**

Part A

Section I

1. HCF of two numbers is 27 and their LCM is 162.
Let the other number be x.
Product of two numbers = HCF × LCM = 27 × 162
⇒ 54x = 27 × 162
⇒ x = 81
⇒ The other number is 81.

OR

Let $x = 0.\overline{8}$ ... (i)
$10x = 8.\overline{8}$ ... (ii)
Subtracting (i) from (ii)
⇒ 9x = 8
⇒ $x = \frac{8}{9}$

2. Mean = 27, median = 33
Mode = 3median – 2mean
⟹Mode = 3 × 33 – 2 × 27
⟹Mode = 45

3. $\sqrt{\frac{1+\sin A}{1-\sin A}}$

$= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$

$= \frac{1+\sin A}{\cos A}$ $\quad \because \sin^2 A + \cos^2 A = 1$

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$

4. Given: $\tan\theta = \frac{a}{b}$

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1 + \tan\theta}{1 - \tan\theta} == \frac{1 + \frac{a}{b}}{1 - \frac{a}{b}} = \frac{b + a}{b - a}$$

5. The distance between the points A(4, p) and B(1, 0) is 5.

$\Rightarrow AB = 5$

$\Rightarrow AB^2 = 25$

$\Rightarrow (4 - 1)^2 + p^2 = 25$

$\Rightarrow 9 + p^2 = 25$

$\Rightarrow p^2 = 16$

$\Rightarrow p = \pm 4$

OR

The point on y-axis, below x-axis, at a distance of 4 units from x-axis is A(0, –4).

6. In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$

$\Longrightarrow \Delta ABC \sim \Delta DEF$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AB)^2}{(DE)^2} = \frac{5^2}{7^2} = \frac{25}{49}$$

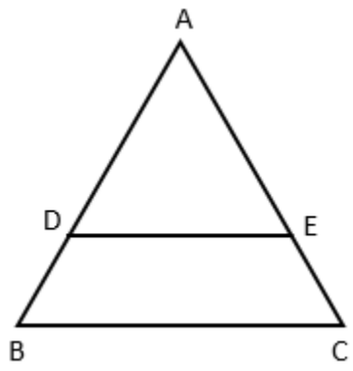
OR

DE || BC

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{4}{7} = \frac{6}{EC}$$

$\Rightarrow EC = 10.5$ cm



7. For an event E, the value of P(E) + P(not E) = 1.

8. From the diagram,

In Δ PTO,

$PT^2 + TO^2 = PO^2$

$24^2 + 7^2 = PO^2$

$PO^2 = 576 + 49$

$PO^2 = 625$

$PO = 25$ cm

9. First n even natural numbers are 2, 4, 6,....2n

$a = 2, a_n = 2n$

$$S_n = \frac{n}{2}(2 + 2n) = n(n + 1)$$

OR

$$\sqrt{8}, \sqrt{18}, \sqrt{32}, ...$$

$$\Rightarrow 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, ...$$

$$\Rightarrow a = 2\sqrt{2}, d = \sqrt{2}, n = 4$$

$$\Rightarrow a_3 = 4\sqrt{2}$$

$$\Rightarrow a_4 = a_3 + d = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$$

10. $7x^2 - 12x + 18 = 0$

$\Rightarrow a = 7, b = -12, c = 18$

Let α, β be the roots of the equation

$$\alpha + \beta = \frac{-b}{a} = \frac{12}{7}$$

$$\alpha\beta = \frac{c}{a} = \frac{18}{7}$$

$$\Rightarrow \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{12}{7}}{\frac{18}{7}} = \frac{12}{18} = \frac{2}{3}$$

11. $3x + 5y = 0$ and $kx + 10y = 0$

Condition for the system of equations to have non-zero solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$a_1 = 3, a_2 = k, b_1 = 5, b_2 = 10$

$$\Rightarrow \frac{3}{k} = \frac{5}{10} \Rightarrow k = 6$$

12. The volume of the given figure is $\frac{1}{3}\pi h\left(r^2 + R^2 + rR\right)$.

13. Let α and β are the roots of the given quadratic equation.

According to the question, $\alpha = 2$ ...(i)

Comparing $x^2 + 3x + k$ with $ax^2 + bx + c$, we get $a = 1, b = 3, c = k$

$\alpha + \beta = -3$ and $\alpha\beta = k$

From (i) $2 + \beta = -3 \Rightarrow \beta = -5$

$\alpha\beta = k \Rightarrow k = -10$

14. Product of zeroes of Cubic Polynomial = $\frac{-d}{a} = \frac{-9}{2}$

$\Longrightarrow$Product of two zeroes × third zero = $\frac{-9}{2}$

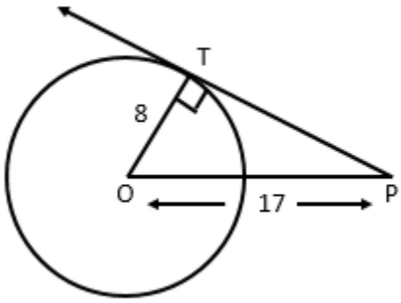
$\Longrightarrow$ third zero = $\frac{-9}{2 \times 3} = \frac{-3}{2}$

15. The word is MOBILE.

The vowels in the word are O, I, E and the total number of words are 6.

The required probability = 3/6 = ½

16. In ΔOTP,



$OP^2 = OT^2 + TP^2$ ∵ OT is perpendicular to the tangent

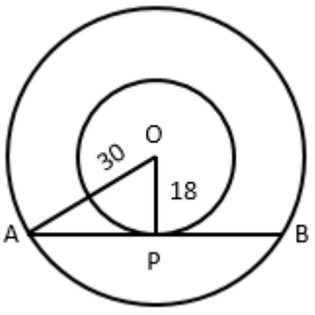
$\Rightarrow TP^2 = OP^2 - OT^2$

$\Rightarrow TP^2 = 17^2 - 8^2$

$\Rightarrow TP^2 = 289 - 64 = 225$

$\Rightarrow TP = 15$ cm

OR



Let O be the centre of the concentric circles of radii 30 cm and 18 cm respectively.
Let AB be a chord of the larger circle touching the smaller circle at P.
So, OP is perpendicular to AB as AP is tangent to the smaller circle at P.
Then AP = PB since OP is perpendicular to AB.
Using Pythagoras theorem in triangle OPA,
$\Rightarrow OA^2 = OP^2 + AP^2$
$\Rightarrow 30^2 = 18^2 + AP^2$
$\Rightarrow AP^2 = 576$
$\Rightarrow AP = 24$ cm
$\Rightarrow AB = 2AP = 48$ cm
Hence, the length of the chord is 48 cm.

Section II

17.

(a) Angles $\angle LKM$ and $\angle JKL$ are called as Linear Pair of angles.
(b) $m\angle LKM + m\angle JKL = 180°$..... Linear Pair
$\Longrightarrow 2x - 15 + m\angle LKM = 180°$
$\Longrightarrow m\angle LKM = 195° - 2x$
(c) In $\triangle LKM$,
$m\angle LKM + m\angle LMK + m\angle KLM = 180°$ ...angle sum property of a triangle
$\Longrightarrow 195° - 2x + 50 + x = 180°$
$\Longrightarrow x = 65° = m\angle KLM$
(d) $m\angle LKM = 195° - 2x = 195 - 2(65) = 195 - 130 = 65°$
In $\triangle LKM$, $m\angle LKM = m\angle KLM = 65°$
$\Longrightarrow \triangle LKM$ is an isosceles triangle.
(e) $m\angle LKJ = 2x - 15 = 2(65) - 15 = 130 - 15 = 115°$

18.

(a) The coordinates of CAVE of DEATH is (5, 3).
(b) The coordinates of THREE PALMS is (6, 4).
(c) The coordinates FOUR CROSS CLIFF and CAVE of DEATH are (2, 3) and (5, 3) respectively.
Distance between them = $\sqrt{(5-2)^2 + (3-3)^2} = \sqrt{9} = 3$ units
(d) The distance of SKULL ROCK from x – axis is 5 units.
(e) The mid – point of CAVE of DEATH and THREE PALMS
$= \left(\frac{5+6}{2}, \frac{3+4}{2}\right) = (5.5, 3.5)$

19.

(a) $x^2 - 18x + 81 = x^2 - 9x - 9x + 81 = x(x - 9) - 9(x - 9) = 0$

$\Rightarrow (x - 9)(x - 9) = 0$

$\Rightarrow x = 9$ or $x = 9$

(b) Zeroes of a polynomial can be expressed graphically. Number of zeroes of a polynomial is equal to the number of points where the graph of polynomial **Intersects x – axis.**

Number of zeroes of the given polynomial(refer image) is equal to 3.

(c) The degree of a Quadratic Polynomial is 2.

(d) A highway underpass is parabolic in shape and a parabola is the graph that results from $p(x) = ax^2 + bx + c$ which has two zeroes (as it is a quadratic polynomial).

Product of zeroes = $6 \times 3 = 18$ and sum of the zeroes = $6 + 3 = 9$

x^2 – (sum of zeroes)x + product of zeroes

$= x^2 - 9x + 18$

(e) $f(m) = m^2 = m^2 + 0m + 0$ is a Quadratic Polynomial.

The number of zeroes that f(x) can have is 2.

20.

(a)

Time (in sec)	No. of students(f)	X	fx
20 – 40	7	30	210
40 – 60	10	50	500
60 – 80	15	70	1050
80 – 100	5	90	450
100 – 120	3	110	330
	$\Sigma f = 40$		$\Sigma fx = 2540$

Mean time taken by a student to finish the race = 2540/40 = 63.5 seconds

(b) The modal class is 60 – 80 as it has the highest frequency i.e 15.

Lower limit of the modal class = 60

(c) Mean, Median and Mode are measures of central tendency.

(d)

Time (in sec)	No. of students(f)	cf
20 – 40	7	7
40 – 60	10	17
60 – 80	15	32
80 – 100	5	37
100 – 120	3	40
	$N = \Sigma f = 40$	

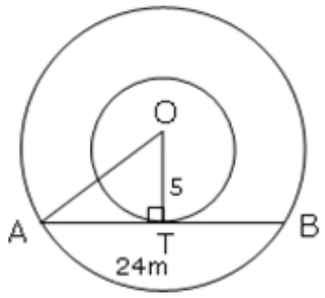
Here N/2 = 40/2 = 20, Median Class = 60 – 80, Modal Class = 60 – 80
Sum of upper limits of median class and modal class = 80 + 80 = 160

(e) Number of students who finished the race within 1 min = 7 + 10 = 17

Part B

Section III

21. Let O be the centre of concentric circles.
AB be the chord of larger circle and OT be the radius of smaller circle.



So OT $\perp$ AB since tangent is $\perp$ to radius at its point of contact.
AT = TB = 12 m
(Since perpendicular from centre to the chord bisects it)
So, in ΔOAT,
$OA^2 = OT^2 + AT^2$
$OA^2 = 5^2 + 12^2 = 169$
$\Rightarrow$ OA = 13 cm
Thus, the radius of the larger circle is 13 cm.

22. Let n be the required number of spheres.
Since, the spheres are melted to form a cylinder.
So, the volume of all the n spheres will be equal to the volume of the cylinder.

$$n \times \frac{4}{3} \times \pi \times 3 \times 3 \times 3 = \pi \times 2 \times 2 \times 45$$

$\therefore$ n = 5
Thus, the required number of spheres which are melted to form the cylinder is 5.

OR

Let x cm be the edge of the new cube.
Volume of the new cube = Sum of the volumes of three cubes
$x^3 = 3^3 + 4^3 + 5^3$
$x^3 = 27 + 64 + 125$
$x^3 = 216$
x = 6.
Edge of the new cube is 6 cm long.

23. Given sides of a triangle are 9 cm, 18 cm, and 16 cm.
Consider, $9^2 + 16^2 = 81 + 256 = 337 \neq 18^2$.
Hence, these sides cannot form right triangle.

OR

The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.
Let a be the area of smaller triangle and A be the area of the larger triangle.

$$\frac{a}{A} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\Rightarrow \frac{48}{A} = \frac{4}{9} \qquad \because \text{Area of smaller triangle} = 48 \text{ cm}^2$$

$\Rightarrow A = 108 \text{ cm}^2$

24. There are 10 ribs in an umbrella.

The area between two consecutive ribs subtends an angle of $\frac{360°}{10} = 36°$ at the centre of the assumed flat circle.

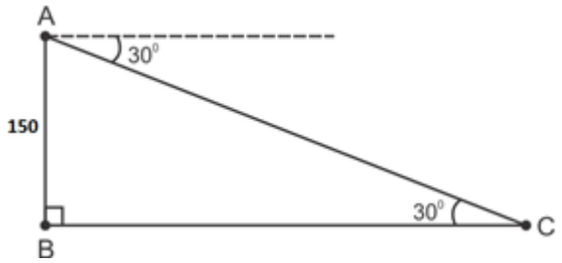
Area between two consecutive ribs of circle $= \frac{36°}{360°} \times \pi r^2$

$$= \frac{36°}{360°} \times \frac{22}{7} \times (40)^2$$

$$= \frac{1}{10} \times \frac{22}{7} \times 40 \times 40$$

$$= 502.86 \text{ cm}^2$$

25. Let AB be the tower and BC be distance between tower and car.



In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{150}{BC}$$

$$\therefore BC = 150\sqrt{3} \text{ m}$$

Hence, distance between the tower and car is $150\sqrt{3}$ m.

26. According to the question,

$n(S) = 20$

1. Let A be an event of getting a number divisible by 2 and 3.

$\therefore A = \{6, 12, 18\}$

$\therefore n(A) = 3$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{20}$

2. Let B be an event of getting a prime number.

$\therefore B = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$\therefore n(B) = 8$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{8}{20} = \frac{2}{5}$

Section IV

27. $(a - b)x^2 + (b - c)x + (c - a) = 0$

The given equation will have equal roots, if

$(b - c)^2 - 4(a - b)(c - a) = 0$

$b^2 + c^2 - 2bc - 4(ac - bc - a^2 + ab) = 0$

$b^2 + c^2 + 4a^2 + 2bc - 4ab - 4ac = 0$

$(b + c - 2a)^2 = 0$

$b + c - 2a = 0$

$b + c = 2a$

28. $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots\ldots \tan 89^\circ$

$= \tan(90^\circ - 89^\circ) \tan(90^\circ - 88^\circ) \tan(90^\circ - 87^\circ) \ldots\ldots\ldots \tan 87^\circ \tan 88^\circ \tan 89^\circ$

$= \cot 89^\circ \cot 88^\circ \cot 87^\circ \ldots\ldots\ldots \tan 87^\circ \tan 88^\circ \tan 89^\circ$

$= \cot 89^\circ \tan 89^\circ \cot 88^\circ \tan 88^\circ \cot 87^\circ \tan 87^\circ \ldots\ldots \cot 44^\circ \tan 44^\circ \tan 45^\circ$

$= 1 \times 1 \times 1 \ldots \times 1 = 1$

$\Rightarrow \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots\ldots \tan 89^\circ = 1$

29. Let $6 + \sqrt{3}$ be rational and equal to $\frac{a}{b}$.

Then, $\frac{6 + \sqrt{3}}{1} = \frac{a}{b}$, where a and b are co primes, $b \neq 0$

$\therefore \sqrt{3} = \frac{a}{b} - 6 = \frac{a - 6b}{b}$

Here a and b are integers.

So, $\frac{a - 6b}{b}$ is rational.

Therefore, $\sqrt{3}$ is rational.

This is a contradiction as $\sqrt{3}$ is irrational.

Hence, our assumption is wrong.

Thus, $6 + \sqrt{3}$ is an irrational number.

OR

Let us assume, on the contrary that $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

Where a and b are co-prime integers.

$$\sqrt{5} = \frac{a}{b} \Rightarrow a = \sqrt{5}b \Rightarrow a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 then a is also divisible by 5.

So a = 5k, for some integer k.

$$Now, a^2 = (5k)^2 = 5(5k^2) = 5b^2$$

$$\Rightarrow b^2 = 5k^2$$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

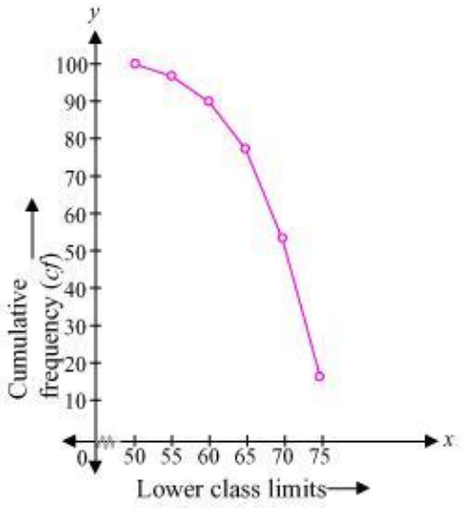
So our assumption that $\sqrt{5}$ is rational is wrong.

Hence, $\sqrt{5}$ cannot be a rational number. Therefore, $\sqrt{5}$ is irrational.

30. We can obtain cumulative frequency distribution of more than type as following:

Production yield (lower class limits)	Cumulative frequency
More than or equal to 50	100
More than or equal to 55	100 - 2 = 98
More than or equal to 60	98 - 8 = 90
More than or equal to 65	90 - 12 = 78
More than or equal to 70	78 - 24 = 54
More than or equal to 75	54 - 38 = 16

Now, taking lower class limits on x-axis and their respective cumulative frequencies on y-axis, we can obtain the ogive as follows:



31.

$$\text{L.H.S.} = \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = \frac{\left(\sqrt{\sec\theta - 1}\right)^2 + \left(\sqrt{\sec\theta + 1}\right)^2}{\left(\sqrt{\sec\theta + 1}\right)\left(\sqrt{\sec\theta - 1}\right)}$$

$$= \frac{\sec\theta - 1 + \sec\theta + 1}{\sqrt{\sec^2\theta - 1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta}$$

$$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta = \text{R.H.S.}$$

OR

$$\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$$

$$= \frac{\sin(90^\circ - 70^\circ)}{\sin 20^\circ} + \frac{\sin(90^\circ - 59^\circ)}{\sin 31^\circ} - 8\left(\frac{1}{2}\right)^2$$

$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 8 \times \frac{1}{4} = 1 + 1 - 2 = 0$$

Since, $\cos\theta = \sin(90^\circ - \theta)$

32. Let a − d, a and a + d be three terms in A.P.

According to the question,

a − d + a + a + d = 3

3a = 3 ⇒a = 1

(a − d) (a) (a + d) = −8

$a(a^2 - d^2) = -8$

Putting the value of a = 1, we get,

$1 - d^2 = -8 \Longrightarrow d^2 = 9$ or d = ±3

Thus, the required three terms are −2, 1, 4 or 4, 1, −2

33.

CI	50-60	60-70	70-80	80-90	90-100	100-110	Total
f_i	5	3	4	p	2	13	27 + p
x_i	55	65	75	85	95	105	
$f_i x_i$	275	195	300	85p	190	1365	2325+ 85p

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow 86 = \frac{2325 + 85p}{27 + p}$$

$$\Rightarrow 86p + 2322 = 2325 + 85p$$

$$\Rightarrow p = 3$$

Section V

34. Steps of construction :

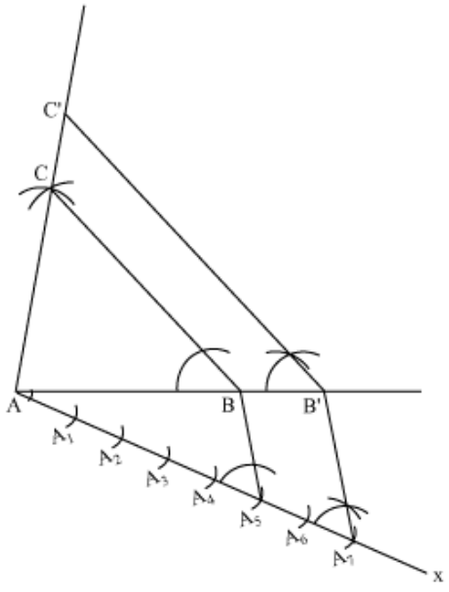
i. Draw a line segment AB of 5 cm. Taking A and B as centres, draw two arcs of 6 cm and 7 cm radius respectively. Let these arcs intersect each other at point C. ΔABC is the required triangle having lengths of sides as 5 cm, 6 cm and 7 cm respectively.

ii. Draw a ray AX making acute angle with the line AB on opposite side of vertex C.

iii. Locate 7 points A_1, A_2, A_3, A_4, A_5, A_6, A_7 (as 7 is greater between 5 and 7) on line AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

iv. Join BA_5 and draw a line through A_7 parallel to BA_5 to intersect extended line segment AB at point B'.

v. Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. ΔAB'C' is the required triangle.

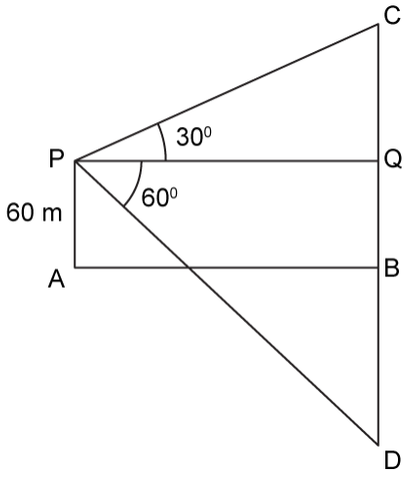


35. Let C be the cloud and D be its reflection. Let the height of the cloud be H metres.

BC = BD = H

BQ = AP = 60 m.

Therefore CQ = H − 60 and DQ = H + 60



In ΔCQP,

$\frac{PQ}{CQ} = \cot 30°$

$\Rightarrow \frac{PQ}{H-60} = \sqrt{3}$

$\Rightarrow PQ = (H-60)\sqrt{3}$ m ....(i)

In ΔDQP,

$\frac{PQ}{DQ} = \cot 60°$

$\Rightarrow \frac{PQ}{H+60} = \frac{1}{\sqrt{3}}$

$\Rightarrow PQ = \frac{(H+60)}{\sqrt{3}}$ .....(ii)

From (i) and (ii),

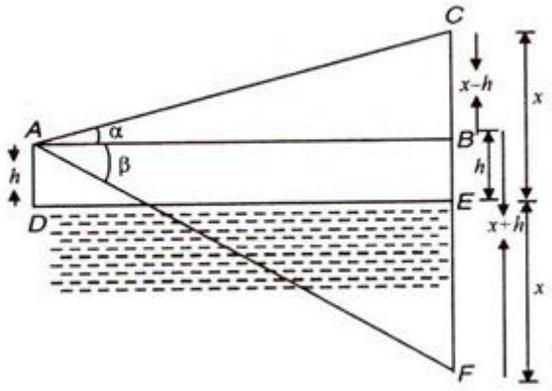
$(H-60)\sqrt{3} = \frac{(H+60)}{\sqrt{3}}$

$\Rightarrow 3H - 180 = H + 60$

$\Rightarrow H = 120$

Thus, the height of the cloud is 120 m.

OR



Let height of the cloud above water level be = x

Then, BC = x – h and BF = x + h

$\frac{BC}{AB} = \tan\alpha$

$\Rightarrow \quad x - h = AB \tan\alpha \quad ...(i)$

and $\quad x + h = AB \tan\beta \quad ...(ii)$

Dividing (i) by (ii), $\frac{x-h}{x+h} = \frac{\tan\alpha}{\tan\beta}$,

use Componendo – Dividendo to get x

$$\frac{x-h+x+h}{x-h-x-h} = \frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$$

$$\frac{2x}{-2h} = \frac{(\tan\beta+\tan\alpha)}{-(\tan\beta-\tan\alpha)} \text{ or } x = h\frac{(\tan\beta+\tan\alpha)}{(\tan\beta-\tan\alpha)}$$

36. Diameter of graphite = 1 mm = 0.1 cm

Therefore, radius of graphite = $\frac{0.1}{2}$ = 0.05 cm

Length of pencil = 10 cm

Volume of graphite = $\pi r^2 h = \frac{22}{7} \times (.05)^2 \times 10 = 0.0785 \text{ cm}^3$

Therefore, weight of graphite = volume × density = 0.0785 × 2.3 = 0.180 gm

Diameter of the pencil = 0.7 cm

Therefore, radius of the pencil = 0.35 cm

Therefore, volume of the pencil = $\pi R^2 h = \frac{22}{7} \times (0.35)^2 \times 10 = 3.85 \text{ cm}^3$

Therefore, volume of wood = Volume of pencil – Volume of graphite

= (3.85 – 0.0785) cm^3

= 3.771 cm^3

Weight of wood = Volume × density = 3.771 × 0.6 = 2.2626 gm